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THEORY AND MODELING OF THE INTERFACIAL TENSION FORCE ON THE OIL SPREADING UNDER ICE COVERS

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ABSTRACT

Interfacial tension force which acts an oil slick on the edge had been believed to be derived from oil-water, ice-water and ice-oil interfacial tensions and the contact angle, but its precise formulation had not been known. In this paper, the interfacial tension force were investigated theoretically and the accurate model of this force was constructed. To verify this model, oil-spill experiments were carried out at the ice model basin at National Maritime Research Institute, to observe the behavior of oil under ice covers. Theoretical estimation of the oil slick size was also carried out. These results meet very well each other.

INTRODUCTION

The risk of oil pollution has been arisen in the Sea of Okhotsk from the start of the crude-oil/gas production at the northern Sakhalin offshore. The production in winter season is not yet carried out, but is planned to start from 2004. Therefore the oil spill in the frozen ocean is not a groundless apprehension. However, research and preparation to cope with this serious situation seems not enough in Japan.

To take countermeasures against that, the research project was organized by five research institutes, including the authors, and is working to figure out the behavior and method of withdrawal of spilled oil in a frozen ocean. We are in charge of theoretical analyses and fundamental experiments. This study is a part of the research project.

If the oil spills under flat ice cover, the oil forms a round slick, thus the slick size can be represented by its radius. The relationship between the interfacial tension and the final slick radius was discussed by Yapa and Chowdhury (1989). They introduced "the net interfacial tension" σ_n, which they considered to be derived from oil/water, ice/water, and ice/oil interfacial tensions and the contact angle. They also carried out the laboratory experiment, and found that these estimation met the experimental results. However, the physical meaning of the net interfacial tension and how to estimate it had not been given.

Konno and Izumiyama (2002) applied the theory of ADSA (Axisymmetric Drop Shape Analysis) to the analysis of Yapa and Chowdhury (1989), and found that the net interfacial tension is twice the oil/water interfacial tension, under the assumption that the contact anble of the oil/ice/water interface is nearly equal to 180°.

THEORY AND APPLICATION OF THE ADSA

In this study we made use of the theory of ADSA (Axisymmetric Drop Shape Analysis) constructed by Rotemberg *et al.* (1983). They introduced the reasonable assumptions that external force is absent, other than gravity, and that the sessile/pendant drop is axisymmetric. Under these assumptions the meridian curve of a sessile/pendant drop can be represented by the arc length s measured from the apex, as $x = x(s)$ and $z = z(s)$, and that obeys the following differential equations:

$$\begin{aligned} \frac{d\bar{x}}{d\bar{s}} &= \cos\phi \\ \frac{d\bar{z}}{d\bar{s}} &= \sin\phi \\ \frac{d\phi}{d\bar{s}} &= 2 + \frac{(\Delta\rho)g\,R_0^2}{\sigma}\bar{z} - \frac{\sin\phi}{\bar{x}}. \end{aligned} \tag{1}$$

Here $\Delta\rho$ is the density difference between the drop and the surrounding fluid, R_0 is the radius of curvature at the apex, σ is the interfacial tension, g is the gravity acceleration, and ϕ is the turning angle measured between the tangent to the interface at the point (x, z) and the datum plane. x, z and s are non-dimensionalized by R_0, as $\bar{s} = s/R_0$, $\bar{x} = x/R_0$ and $\bar{z} = z/R_0$. $(\Delta\rho)gR_0^2/\sigma$ is the shape parameter and is often written as β. It should be noted that the above equations represent both pendant and sessile drops, depending on the axis arrangement and the choice of plus and minus of $\Delta\rho$.

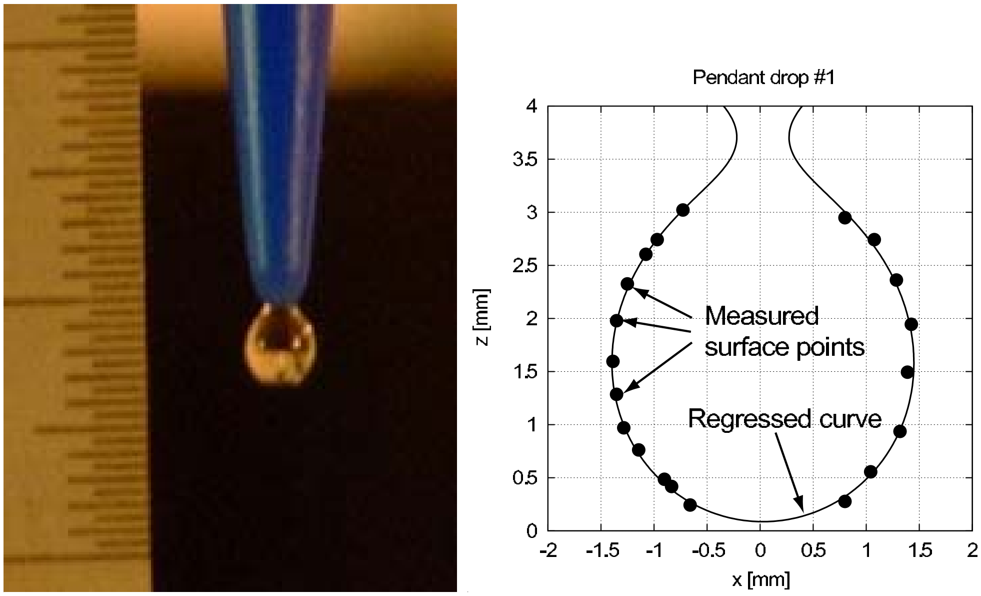
The above differential equations (1) determine the drop shape with given R_0 and $\beta = (\Delta\rho)gR_0^2/\sigma$. To determine the interfacial tension σ from the drop shape, the curve-fitting problem must be solved with parameters R_0 and β to fit the calculated drop shape to the measured one. Solution method for the differential equations and optimization method used in this research were explained by Konno and Izumiyama (2002), and are not explained here for the lack of space.

Verification of the method

To verify the method, measurement and related programs, the surface tension of the water (the interfacial tension of the water/air interface) was measured by the pendant-drop method.

Figure 1 shows the shape of the water drop and measured results of this drop. Measured surface tension was 99.7 mJ/m^2, at 4 °C while the 'real' surface tension is 74.90 mJ/m^2 at 5 °C (National Astronomical Observatory, 2000).

Considering the difficulties of the measurement of the surface tension of the water, because of its high sensitivity to the surfactant additives, this result looks reasonable. Further verification is in progress.



$R_0 = 1.38\,\mathrm{mm}$, $\sigma = 99.7\,\mathrm{mJ/m^2}$

Figure 1: An example of the ADSA measurement: water in the air for verification

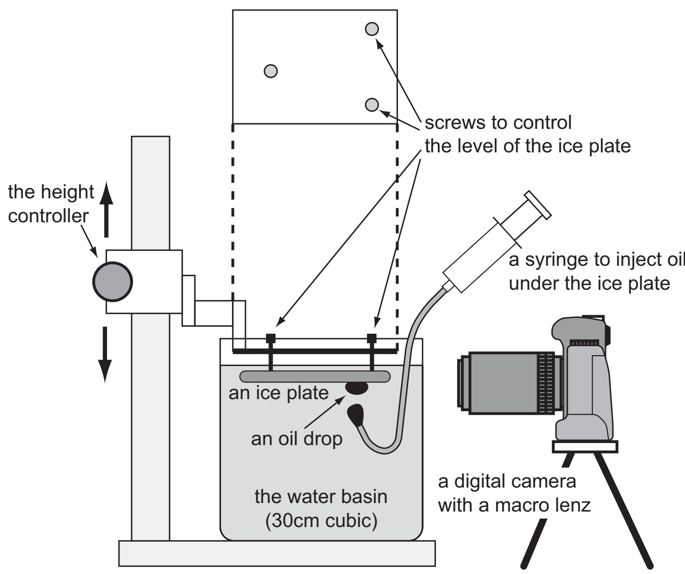
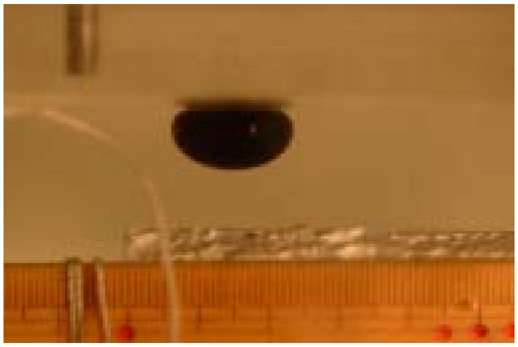
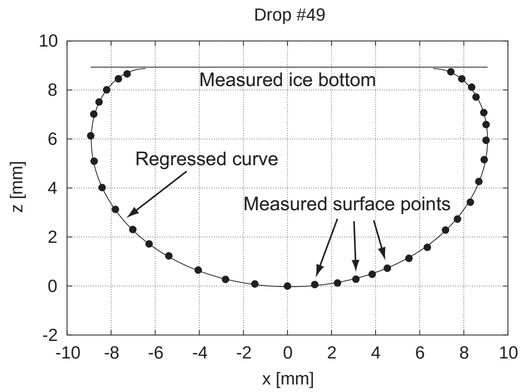


Figure 2: Experimental apparatus to measure the oil drop shapes

Measurement of the oil/water interfacial tension

Figure 2 shows the experimental apparatus to measure the meridian shape of the oil drop. Mechanical oil #10 was provided for the measurement, after blackened by oil-solvent dye. The size of drops was varied so that drops of different shapes were measured.

Examples of the drop shapes and measured results are shown in Figure 3. The average of the oil/water interfacial tension of this oil was $0.0262\,\mathrm{J/m^2}$.

$R_0 = 14.0\,\mathrm{mm}$, $\sigma = 25.6\,\mathrm{mJ/m^2}$

Figure 3: An example of the ADSA measurement: oil in the water

The flat lines in the lower graphs of Figure 3(a) and (b) represent measured bottoms of the ice sheets. To measure the contact angle, the differential equations (1) is integrated until $z = R_0\bar{z}$ reaches the ice sheet. ϕ at that time is the contact angle. In this measurement, however, the curvature sometimes did not reach the line before ϕ reached 180°, as shown in Figure 3(a). That was because it was hard to accurately determine the ice bottom from photographs such as Figure 3. From limited results of the measurement, we concluded that the contact angle was near 180°.

Further explanation and measurement results are reported in Konno and Izumiyama (2002).

MODELING OF THE INTERFACIAL TENSION FORCE FOR THE OIL SPILL PROBLEM

If the oil spills under ice cover, the oil slick may behave under the effect of buoyancy, interfacial tensions between oil/water and oil/ice interfaces, friction from water and ice, and adhesion to the ice bottom. However, these important parameters were not yet formulated clearly, except the buoyancy. In this section we make the model equation of the interfacial tension force. The following matters are assumed: (1) the bottom of the oil slick is flat, except near the edge, and the edge of the oil slick is smooth, (2) the thickness of the oil slick h is by far smaller than the radius R, and (3) contact angle on oil/water/ice interface is 180°.

We start from the classical Laplace equation which describes the pressure difference across a curved interface:

$$\sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \Delta P, \tag{2}$$

where σ is the interfacial tension, R_1 and R_2 represent the two principle radii of curvature, and ΔP is the pressure difference across the interface. By using the radius of curvature on the edge $r = r(s)$, which is defined as shown in Figure 4, Eq.(2) can be transformed as follows:

$$\Delta P = \sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \approx \frac{\sigma}{r}. \qquad (O(r/R_2) = O(h/R) \ll 1)$$

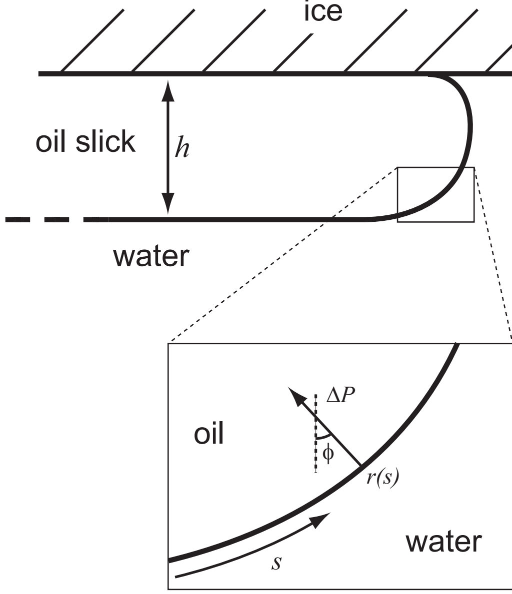


Figure 4: The illustration of the edge shape of the oil slick and relationship between the position, the radius of curvature and the perpendicular direction of the surface

In addition, from the geometrical relationship, $d\phi/ds = 1/r$. Therefore,

$$\Delta P = \sigma \frac{d\phi}{ds}.$$

To get the interfacial tension force on a unit length of the slick edge f_t, the levelly component of this pressure difference should be integrated.

$$\begin{aligned} f_t &= \int_{\mathrm{arc}} \Delta P \sin\phi \, ds = \int_{\mathrm{arc}} \sigma \frac{d\phi}{ds} \sin\phi \, ds \\ &= \int_0^{\pi} \sigma \sin\phi \, d\phi = 2\sigma \end{aligned} \tag{3}$$

Here we have got the simple model: $f_t = 2\sigma$.

It should be mentioned that the upper limit of the interval of integration in Eq.(3) is equal to the contact angle. Therefore the net interfacial tension and the final slick radius are functions of the oil/ice interfacial tension and the contact angle.

Verification of the model by theoretical and experimental results

To verify this model by theory and experiments, the approximate relationship between volume and radius of the oil slick under flat ice covers will be formulated. Suppose the oil slick is large and flat so that the volume V can be approximated to be the product of the thickness and the bottom area, as $V = \pi R^2 h$. The relationship between the oil thickness h and the interfacial tension force on a unit length f_t comes from the consideration of the balance of buoyancy and f_t, under assumption of hydrostatic conditions, and yields

$$\frac{1}{2}(\Delta\rho) g h^2 = f_t.$$

From the above equations and Eq. (3), h is eliminated and the relationship between V and R is derived, as follows.

$$\frac{1}{2}(\Delta\rho)g\left(\frac{V}{\pi R^2}\right)^2 = 2\sigma$$

$$\Leftrightarrow R = \left(\frac{(\Delta\rho)gV^2}{4\pi^2\sigma}\right)^{1/4}. \tag{4}$$

On the other hand, if the radius of curvature at the apex R_0 are given, the shape of the corresponding oil drop (slick) can be calculated by integrating Eqs.(1) from $s = 0$ until $\phi = 180°$. The radius of the slick is the maximum value of $x = R_0\bar{x}$. The volume of the slick can also be calculated by again integrating the shape of the meridian. By varying R_0 parametrically, the relationship between the volume and the radius of the oil slick is obtained. This relationship was adopted as the theoretical relationship.

We carried out the oil-spill experiments using the ice model basin at National Maritime Research Institute. Experimental facilities, conditions and results are reported in Izumiyama and Konno (2002). In this paper we limit ourselves to a consideration of the results of the experiments under flat ice covers to compare with the model and theoretical relationship.

Figure 5 shows the measured results, theoretical estimation and the model estimation of the oil slick radii. It is obvious that the theoretical line ('Theory' in Figure 5) and model line ('Model') are very close each other. And these meet the experimental result ('Experiment' in Figure 5) very well, especially in the small volume region. In the large volume region there are small discrepancies and the experimental results are smaller than the theoretical estimation. It might be because the radii of the oil slicks were not yet converged so that these were smaller than the final slick radii.

Application of the model to the oil spill under uneven ice covers

The above discussion deals with oil spills under flat ice covers. For oil spills under uneven ice covers, the above estimations are not appropriate, but the model of the interfacial tension force can be applied.

The model equation says that the interfacial tension force is twice of the oil/ice interfacial tension par a unit length. Figure 6 illustrates the interfacial tension acting an oil slick on the interface. The strength of the interfacial tension force is a function only of the length of the interface, and the direction is always from outside to inside, normal to the interface. So we should only consider the two-dimensional problem, and the key question is how to determine the boundary of the oil slick.

Numerical modeling and simulation of the oil spill under uneven ice covers are in progress as a part of our research project.

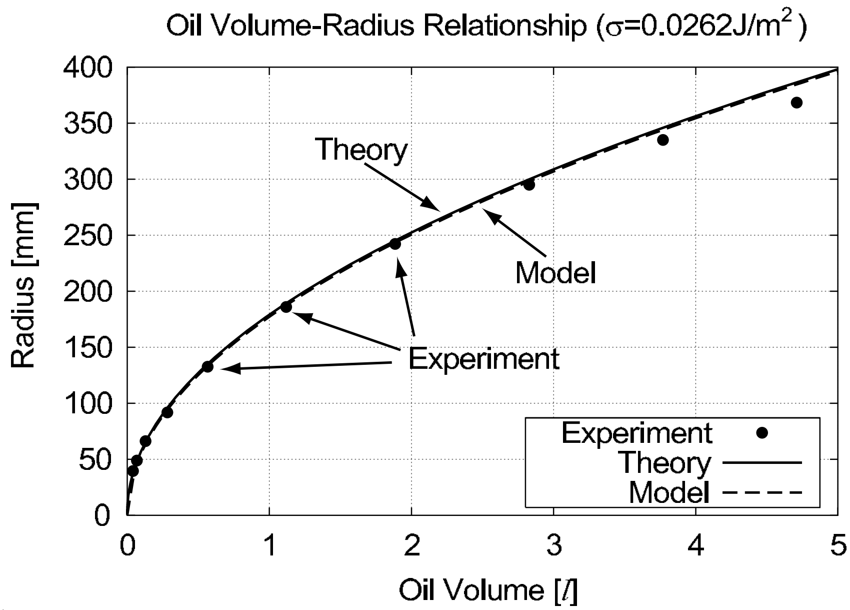


Figure 5: Comparison of the theoretical estimation and the experimental results of the oil slick radii

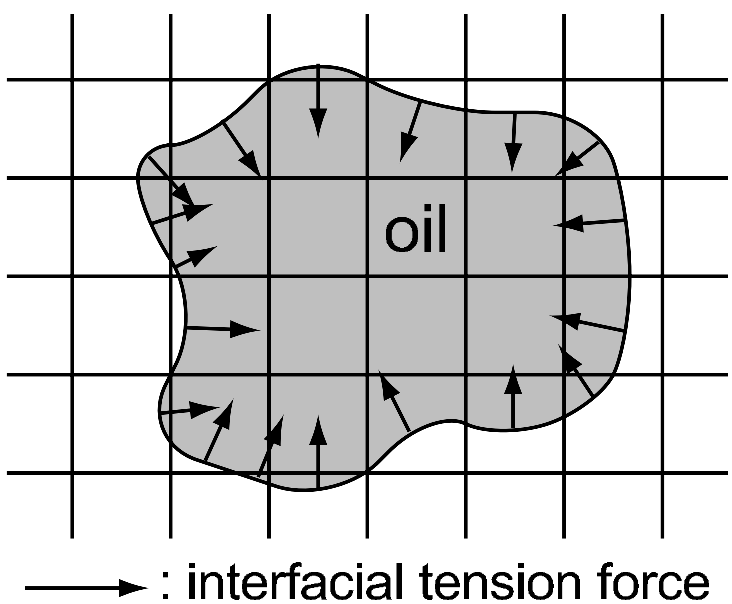


Figure 6: An illustration of the interfacial tension force acting on an oil slick

CONCLUSIONS

Theoretical analysis were carried out to clarify the relationship between the oil/water interfacial tension and the oil/water/ice contact angle, and the final slick radius. The results were verified by the experiments. The following conclusions were obtained.

1. The oil/water interfacial tension was successfully measured by the ADSA method.
2. A simple but accurate model equation of the interfacial tension acting an oil slick on the edge is constructed: $f_t = 2\sigma$.
3. The final slick radius under flat ice cover can be accurately estimated using the above model equation.
4. Application to the oil spill problem under uneven ice covers is discussed.

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