

関数計算と多項式補間

No.

ニ等ノ補間ニシテ計算

x_i	f_i	b_0					
0.0	1.0						
0.5	0.87758		$\frac{0.87758 - 1.0}{0.5} = -0.24484$	b_1			
1.0	0.54030		$\frac{0.54030 - 0.87758}{0.5} = -0.67456$		$\frac{-0.42972}{0.5} = -0.85944$	b_2	
1.5	0.07073		$\frac{0.07073 - 0.54030}{0.5} = -0.93914$		$\frac{-0.26458}{0.5} = -0.52916$		$\frac{0.11009}{0.5} = 0.22018$
2.0	-0.41615		$\frac{-0.41615 - 0.07073}{0.5} = -0.97376$		$\frac{-0.03462}{0.5} = -0.06924$		$\frac{0.15331}{0.5} = 0.30662$
							b_3
							b_4

$$\begin{aligned}
 x = 0.25 \quad y_4 &= b_4 = 0.02161 \\
 y_3 &= b_3 + (x - 1.5)y_4 \\
 &= 0.11009 - 1.25 \times 0.02161 = 0.0830775 \\
 y_2 &= b_2 + (x - 1.0)y_3 \\
 &= -0.42972 - 0.75 \times 0.0830775 = -0.49203 \\
 y_1 &= b_1 + (x - 0.5)y_2 \\
 &= -0.24484 - 0.25 \times (-0.49203) \\
 &= -0.12183 \\
 y &= y_0 = b_0 + (x - 0.0)y_1 \\
 &= 1.0 + 0.25 \times (-0.12183) \\
 &= 0.96954 \quad \dots \left(\frac{7}{6}\right)
 \end{aligned}$$

同様に計算して

$x = 0.75 \rightarrow$	$y = 0.73140$	} $\left(\frac{7}{6}\right)$
$x = 1.25 \rightarrow$	$y = 0.31562$	
$x = 1.75 \rightarrow$	$y = -0.17900$	

数値積分

No. _____

$$\int_{-2}^3 (x^3 - 2x^2 + x - 1) dx$$

(1) 解析解

$$\begin{aligned} \int_{-2}^3 (x^3 - 2x^2 + x - 1) dx &= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - x \right]_{-2}^3 \\ &= \frac{81}{4} - 18 + \frac{9}{2} - 3 - \left(4 + \frac{16}{3} + 2 + 2 \right) \\ &= -\frac{115}{12} \quad \dots \left(\frac{97}{6} \right) \end{aligned}$$

(2) シンプソン則

$$f(x) \equiv x^3 - 2x^2 + x - 1 \text{ とおく}$$

$$f(-2) = -19, \quad f(3) = 11$$

$$f\left(\frac{-2+3}{2}\right) = f\left(\frac{1}{2}\right) = -\frac{7}{8}$$

3点法
-2x²の係数

$$\begin{aligned} \int_{-2}^3 f(x) dx &\approx \frac{3-(-2)}{6} \left\{ f(-2) + 4f\left(\frac{1}{2}\right) + f(3) \right\} \\ &= \frac{5}{6} \left(-19 + 4 \times \left(-\frac{7}{8}\right) + 11 \right) = -\frac{115}{12} \quad \dots \left(\frac{97}{6} \right) \end{aligned}$$

(3) 5分割して台形則

$$f(-1) = -5, \quad f(0) = -1, \quad f(1) = -1, \quad f(2) = 1$$

$$\int_{-2}^3 f(x) dx = \int_{-2}^{-1} f(x) dx + \int_{-1}^0 f(x) dx + \int_0^1 f(x) dx + \int_1^2 f(x) dx + \int_2^3 f(x) dx$$

$$\approx \frac{1}{2}(-19-5) + \frac{1}{2}(-5-1) + \frac{1}{2}(-1-1) + \frac{1}{2}(-1+1) + \frac{1}{2}(1+1)$$

$$= -12 - 3 - 1 + 0 + 1 = -10 \quad \dots \left(\frac{97}{6} \right)$$

線形方程式の解法

$$\begin{array}{cccc|c} 1 & 2 & 1 & -2 & 1 \\ 3 & 1 & -2 & 4 & 3 \\ 1 & -1 & 2 & 3 & 12 \\ -1 & 2 & 3 & 1 & 6 \end{array}$$

↓ピボット選択

$$\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 1 & 2 & 1 & -2 & 1 \\ 1 & -1 & 2 & 3 & 12 \\ -1 & 2 & 3 & 1 & 6 \end{array}$$

↓前進消去

$$\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 0 & \frac{5}{3} & \frac{5}{3} & -\frac{10}{3} & 0 \\ 0 & -\frac{4}{3} & \frac{8}{3} & \frac{5}{3} & 11 \\ 0 & \frac{7}{3} & \frac{7}{3} & \frac{7}{3} & 7 \end{array}$$

↓ピボット選択

$$\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 0 & \frac{7}{3} & \frac{7}{3} & \frac{7}{3} & 7 \\ 0 & -\frac{4}{3} & \frac{8}{3} & \frac{5}{3} & 11 \\ 0 & \frac{5}{3} & \frac{5}{3} & -\frac{10}{3} & 0 \end{array}$$

↓前進消去

$$\begin{array}{cccc|c} 3 & 1 & -2 & 4 & 3 \\ 0 & \frac{7}{3} & \frac{7}{3} & \frac{7}{3} & 7 \\ 0 & 0 & 4 & 3 & 15 \\ 0 & 0 & 0 & -5 & -5 \end{array}$$

↙ 後退代入

$$\begin{array}{cccc|c} 3 & 1 & -2 & 0 & -1 \\ 0 & \frac{7}{3} & \frac{7}{3} & 0 & \frac{14}{3} \\ 0 & 0 & 4 & 0 & 12 \\ 0 & 0 & 0 & -5 & -5 \end{array}$$

↓後退代入

$$\begin{array}{cccc|c} 3 & 1 & 0 & 0 & 5 \\ 0 & \frac{7}{3} & 0 & 0 & -\frac{7}{3} \\ 0 & 0 & 4 & 0 & 12 \\ 0 & 0 & 0 & -5 & -5 \end{array}$$

↓後退代入

$$\begin{array}{cccc|c} 3 & 0 & 0 & 0 & 6 \\ 0 & \frac{7}{3} & 0 & 0 & -\frac{7}{3} \\ 0 & 0 & 4 & 0 & 12 \\ 0 & 0 & 0 & -5 & -5 \end{array}$$

以上より

$$\begin{cases} 3x_1 = 6 \\ \frac{7}{3}x_2 = -\frac{7}{3} \\ 4x_3 = 12 \\ -5x_4 = -5 \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 = 2 \\ x_2 = -1 \\ x_3 = 3 \\ x_4 = 1 \end{cases} \dots \left(\frac{4}{3}\right)$$